1 PLAN AND PREPARE

Warm-Up Exercises

Use a table or graph to solve each equation. Round to the nearest hundredth when necessary.

1. $8^x = 80$ **2.11**
2. $e^{2x} + 5 = 32$ **1.65**
3. $2000(1 - 0.25)^x = 500$ **4.82**
4. $\log 3x = 1.5$ **10.54**
5. $\log_4 x = 3.5$ **128**

2 FOCUS AND MOTIVATE

Essential Question

Big Idea 2, p. 477

How can a graphing calculator be used to solve an exponential or logarithmic inequality? **Tell students they will learn how to answer this question by solving exponential and logarithmic inequalities using both tables and graphs.**

3 TEACH

Extra Example 1

A townhouse was purchased for \$125,000. The value of the townhouse increases by 6% each year. During what interval of time will the townhouse be less than \$175,000? **during about the first 5.77 yr (or 5 yr 9 months) after the townhouse is purchased**

NCTM STANDARDS

Standard 2: Understand functions; Represent situations using algebraic symbols

Extension

Use after Lesson 7.6

Solve Exponential and Logarithmic Inequalities

GOAL Solve exponential and logarithmic inequalities using tables and graphs.

In the Problem Solving Workshop on pages 523–525, you learned how to solve exponential and logarithmic equations using tables and graphs. You can use these same methods to solve exponential and logarithmic inequalities.

EXAMPLE 1 Solve an exponential inequality

CARS Your family purchases a new car for \$20,000. Its value decreases by 15% each year. During what interval of time does the car's value exceed \$10,000?

Solution

Let y represent the value of the car (in dollars) x years after it is purchased. A function relating x and y is $y = 20{,}000(1 - 0.15)^x$, or $y = 20{,}000(0.85)^x$. To find the values of x for which $y > 10{,}000$, solve the inequality $20{,}000(0.85)^x > 10{,}000$.

METHOD 1 Use a table

STEP 1 **Enter** the function $y = 20{,}000(0.85)^x$ into a graphing calculator. Set the starting x-value of the table to 0 and the step value to 0.1.

```
TABLE SETUP
 TblStart=0
 △Tbl=0.1
Indpnt: Auto Ask
Depend: Auto Ask
```

STEP 2 **Use** the *table* feature to create a table of values. Scrolling through the table shows that $y > 10{,}000$ when $0 \le x \le 4.2$.

X	Y1
4	10440
4.1	10272
4.2	10106
4.3	9943.3
4.4	9783

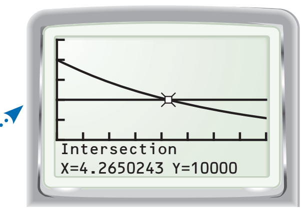
X=4.2

▶ The car value exceeds \$10,000 for about the first 4.2 years after it is purchased.

To check the solution's reasonableness, note that $y \approx 10{,}440$ when $x = 4$ and $y \approx 8874$ when $x = 5$. So, $4 < x < 5$, which agrees with the solution obtained above.

METHOD 2 Use a graph

Graph $y = 20{,}000(0.85)^x$ and $y = 10{,}000$ in the same viewing window. Set the viewing window to show $0 \le x \le 8$ and $0 \le y \le 25{,}000$. Using the *intersect* feature, you can determine that the graphs intersect when $x \approx 4.27$.



The graph of $y = 20{,}000(0.85)^x$ is above the graph of $y = 10{,}000$ when $0 \le x < 4.27$.

▶ The car value exceeds \$10,000 for about the first 4.27 years after it is purchased.

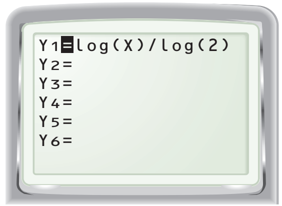
EXAMPLE 2 Solve a logarithmic inequality

Solve $\log_2 x \le 2$.

Solution

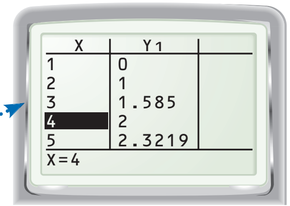
METHOD 1 Use a table

STEP 1 **Enter** the function $y = \log_2 x$ into a graphing calculator as $y = \dfrac{\log x}{\log 2}$.



STEP 2 **Use** the *table* feature to create a table of values. Identify the x-values for which $y \le 2$. These x-values are given by $0 < x \le 4$.

Make sure that the x-values are reasonable and in the domain of the function ($x > 0$).

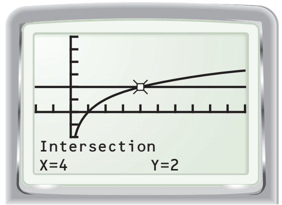


▸ The solution is $0 < x \le 4$.

METHOD 2 Use a graph

Graph $y = \log_2 x$ and $y = 2$ in the same viewing window. Using the *intersect* feature, you can determine that the graphs intersect when $x = 4$.

The graph of $y = \log_2 x$ is on or below the graph of $y = 2$ when $0 < x \le 4$.



▸ The solution is $0 < x \le 4$.

PRACTICE

EXAMPLE 1 on p. 526 for Exs. 1–6

Solve the exponential inequality using a table and using a graph.

1. $3^x \le 20$ **$x \le 2.727$**
2. $28\left(\frac{2}{3}\right)^x > 9$ **$x < 2.799$**
3. $244(0.35)^x \ge 50$ **$x \le 1.51$**
4. $-63(0.96)^x < -27$ **$x < 20.76$**
5. $95(1.6)^x \le 1620$ **$x \le 6.03$**
6. $-284\left(\frac{9}{7}\right)^x > -135$ **$x < -2.96$**

EXAMPLE 2 on p. 527 for Exs. 7–12

Solve the logarithmic inequality using a table and using a graph.

7. $\log_3 x \ge 3$ **$x \ge 27$**
8. $\log_5 x < 2$ **$0 < x < 25$**
9. $\log_6 x + 9 \le 11$ **$0 \le x \le 36$**
10. $2\log_4 x - 1 > 4$ **$x > 32$**
11. $-4\log_2 x > -20$ **$0 < x < 32$**
12. $0 \le \log_7 x \le 1$ **$1 \le x \le 7$**

13. **FINANCE** You deposit \$1000 in an account that pays 3.5% annual interest compounded monthly. When is your balance at least \$1200? **after 5.25 yr**

14. **RATES OF RETURN** An investment that earns a rate of return r doubles in value in t years, where $t = \dfrac{\ln 2}{\ln(1 + r)}$ and r is expressed as a decimal. What rates of return will double the value of an investment in less than 10 years? **$r > 7.18\%$**

Extra Example 2

Solve $\log_3 (x + 2) > 3$. **$x > 25$**

Key Question to Ask for Example 2

- In Example 2, why is the solution $0 < x \le 4$, rather than just $x \le 4$? **The domain of the function $y = \log_2 x$ is $x > 0$, so only positive numbers can be solutions of the given inequality.**

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How can a graphing calculator be used to solve an exponential or logarithmic inequality?

- **To solve an exponential or logarithmic inequality by using a table or graph, use the same basic methods as you would use for solving the equation that would be obtained by replacing the inequality symbol with an equal sign. Then, depending on the inequality symbol, observe the interval of x-values that gives the solution of the inequality.**

A graphing calculator can be used to solve any exponential or logarithmic inequality either by using a table or a graph.

4 PRACTICE AND APPLY

Teaching Strategy

After students have completed the practice exercises, ask them which method they prefer and why. Lead a discussion of the advantages and disadvantages of each method.